3.1 Squares and Square Roots

Focus on...

After this lesson, you will be able to...

- ❑ determine the square of a whole number
- ❑ determine the square root of a perfect square

The Pythagoreans were members of an academy of study that existed 2500 years ago. They created square numbers by arranging pebbles in equal numbers of rows and columns. Nine pebbles could be arranged in three rows and three columns. Nine is a square number because $3 \times 3 = 9$. The picture shows the first four square numbers that the Pythagoreans found: 1, 4, 9, and 16. How can you determine the next square number?

Did You Know?

Pythagoras (about 580–500 B.C.E.) was the leader of a group of academics called the Pythagoreans. They believed that patterns in whole numbers could help explain the universe.

Literacy Link

A *square number* is the product of the same two numbers. 3 x 3 = 9, so 9 is a square number.

A square number is also known as a *perfect square*. A number that is not a perfect square is called a *non-perfect square*.

Materials

- square tiles

Explore the Math

How can you identify a perfect square?

1. Use square tiles to make five rectangles with the dimensions shown.

What is the area of each rectangle?

Length (cm)	Width (cm)
5	3
8	2
9	1
4	3
9	4

2. Try to rearrange the tiles in each rectangle to make a square.

a) Which rectangles can you make into squares?

b) What is the side length of each square?

c) How is the area of each square related to its side length?

3. a) Choose three perfect squares and three non-perfect squares.

b) Express each number as a product of prime factors.

c) For each number, how many times does each prime factor appear? Compare your results with a partner's results.

Literacy Link

Prime Numbers and Prime Factors

A *prime number* is a whole number greater than 1 that has only two factors: 1 and itself.

Prime factors are factors that are prime numbers.

For example, the prime factors of 10 are 2 and 5.

4. a) What do all of the perfect squares have in common?

b) What do all of the non-perfect squares have in common?

Reflect on Your Findings

5. a) How can square tiles help you to determine if a number is a perfect square?

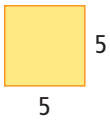
b) How can prime factors help you to determine if a number is a perfect square?

prime factorization

- a number written as the product of its prime factors
- the prime factorization of 6 is 2×3

perfect square

- a number that is the product of the same two factors
- has only an even number of prime factors
- $5 \times 5 = 25$, so 25 is a perfect square



Example 1: Identify Perfect Squares

a) Determine the **prime factorization** of the following numbers: 24, 36, 81.

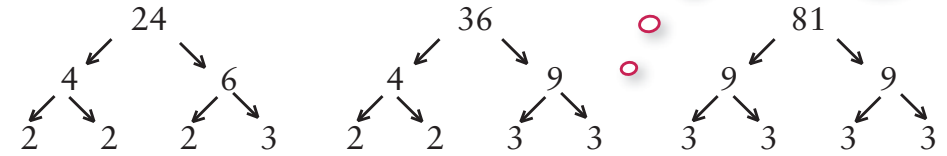
b) Which of the numbers is a **perfect square**? Explain.

c) For each number that is a perfect square, draw the square and label its side length.

Different factor trees are possible to arrive at the same prime factorization.

Solution

a)



$24 = 2 \times 2 \times 2 \times 3$ $36 = 2 \times 2 \times 3 \times 3$ $81 = 3 \times 3 \times 3 \times 3$

b) To be a perfect square, each prime factor in the prime factorization must occur an even number of times. 36 and 81 are perfect squares because each prime factor occurs an even number of times.

$36 = 2 \times 2 \times 3 \times 3$ two factors of 2, two factors of 3

$81 = 3 \times 3 \times 3 \times 3$ four factors of 3

24 is not a perfect square because at least one of the prime factors occurs an odd number of times.

$24 = 2 \times 2 \times 2 \times 3$ three factors of 2, one factor of 3

c) To determine the side length of the squares, look at the product of prime factors for the area.

$36 = 2 \times 2 \times 3 \times 3$ $81 = 3 \times 3 \times 3 \times 3$

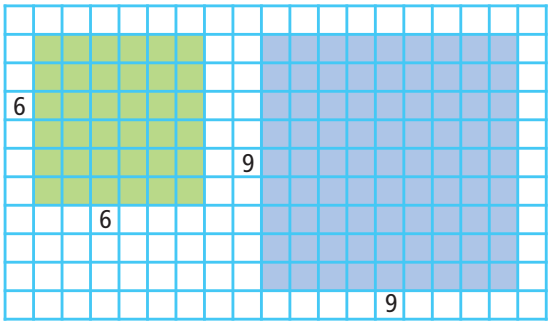
Rearrange the prime factors into two equal groups.

$36 = (2 \times 3) \times (2 \times 3)$

$36 = 6 \times 6$

$81 = (3 \times 3) \times (3 \times 3)$

$81 = 9 \times 9$



Show You Know

Write the prime factorization of each number. Which number is not a perfect square? Explain how you know.

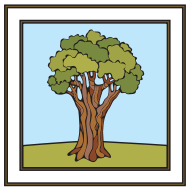
a) 45 **b)** 100

Example 2: Determine the Square of a Number

Determine the area of a square picture with a side length of 13 cm.

Solution

$A = s^2$
$A = 13^2$
$A = 13 \times 13$
$A = 169$
The area is 169 cm^2.



area of a square = side length × side length
$A = s \times s$
$A = s^2$

Strategies
Draw a Diagram

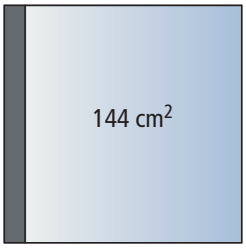
Literacy Link
You can write a repeated multiplication like 13×13 as a square: $13 \times 13 = 13^2$.
13^2 is read as thirteen squared.

Show You Know

Determine the area of a square with a side length of 16 mm.

Example 3: Determine the Square Root of a Perfect Square

Edgar knows that the square case for his computer game has an area of 144 cm^2. What is the side length of the case?



Solution

Method 1: Use Inspection

To find the side length, determine what positive number when multiplied by itself equals 144.
$12 \times 12 = 144$
The **square root** of 144 is 12, or $\sqrt{144} = 12$.
The side length is 12 cm.

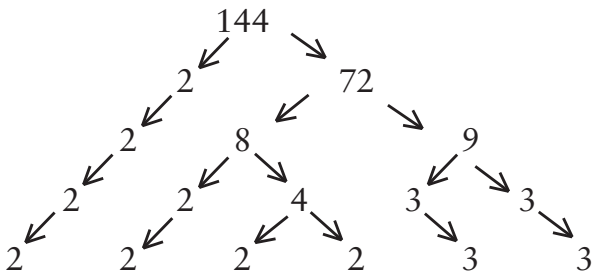
square root
- a number that when multiplied by itself equals a given value
- 6 is the square root of 36 because $6 \times 6 = 36$

Method 2: Use Guess and Check

Find the positive value for the blank boxes.
■ × ■ = 144
$10 \times 10 = 100$ Too low
$13 \times 13 = 169$ Too high
$12 \times 12 = 144$ Correct!
$12 = \sqrt{144}$
The side length is 12 cm.

Literacy Link
Reading Square Roots
The symbol for square root is $\sqrt{\ }$.
Read $\sqrt{9}$ as the square root of 9, square root 9, or root 9.

Method 3: Use Prime Factorization



The prime factorization of 144 is $2 \times 2 \times 2 \times 2 \times 3 \times 3$.

Rearrange the prime factors into two equal groups.

$144 = (2 \times 2 \times 3) \times (2 \times 2 \times 3)$

$144 = 12 \times 12$

$\sqrt{144} = 12$

The side length is 12 cm.

Tech Link

You can use a calculator to find the square root of a number. Try the following key sequences on your calculator. Then, record the one that works on your calculator.

C 144 √ =

or

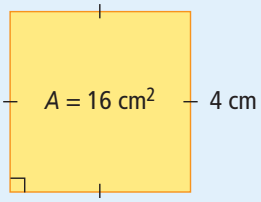
C √ 144 =

Show You Know

Determine the side length of a square with an area of 196 cm^2.

Key Ideas

- The square of a number is the number multiplied by itself.
  $5 \times 5 = 25$, or $5^2 = 25$
- The square of a whole number is a perfect square. $2^2 = 4$
  So, 4 is a perfect square.
- The square of a number can be thought of as the area of a square.
  $4^2 = 16$
  The area is 16 cm^2.

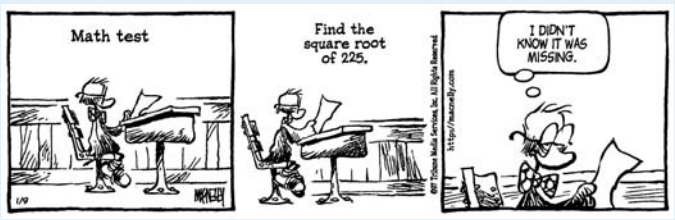


- The square root of a number can be thought of as the side length of a square.
  $\sqrt{16} = 4$
  The side length is 4 cm.
- The square root of a value is a number that when multiplied by itself equals the value.
  $6 \times 6 = 36$, so $\sqrt{36} = 6$
- In the prime factorization of a perfect square, there is an even number of each prime factor.
  $36 = 2 \times 2 \times 3 \times 3$ two factors of 2, two factors of 3

Communicate the Ideas

1. Explain how to square the number 7.

2. How would you use prime factorization to determine the square root of 225? Compare your answer with a classmate's.



3. The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. Use words and/or diagrams to explain how you know which factor is the square root of 36.

4. Explain how squaring a number is the reverse of finding the square root of a number. Include an example with your explanation.

Check Your Understanding

Practise

For help with #5 to #8, refer to Example 1 on page 82.

5. **a)** Determine the prime factorization of 4.
b) Is 4 a perfect square? Explain.
c) Draw the square and label its side length.

6. A rectangle has an area of 64 m^2.
a) Determine the prime factorization of 64.
b) Is 64 a perfect square? Explain.
c) Draw a square with that area and label its side length.

7. Write the prime factorization of each number. Identify the perfect squares.
a) 42 **b)** 169 **c)** 256

8. Determine the prime factorization of each number. Which numbers are perfect squares?
a) 144 **b)** 60 **c)** 40

For help with #9 to #12, refer to Example 2 on page 83.

9. What is the area of a square with each side length?
a) 10 **b)** 16

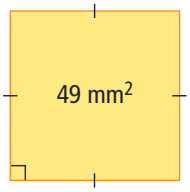
10. Determine the area of a square with each side length.
a) 20 **b)** 17

11. What is the square of each number?
a) 9 **b)** 11

12. Determine the square of each number.
a) 3 **b)** 18

For help with #13 to #16, refer to Example 3 on pages 83–84.

13. What is the side length of the square shown?



14. Determine the side length of a square with an area of 900 cm^2.

15. Evaluate.

a) $\sqrt{49}$ **b)** $\sqrt{64}$ **c)** $\sqrt{625}$

16. Determine the value.

a) $\sqrt{9}$ **b)** $\sqrt{25}$ **c)** $\sqrt{1600}$

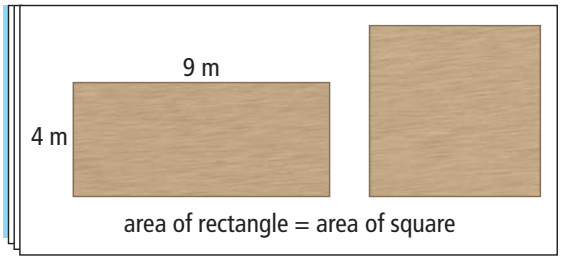
Apply

17. A fridge magnet has an area of 54 mm^2. Is 54 a perfect square? Use prime factorization to find the answer.

18. A floor mat for gymnastics is a square with a side length of 14 m. What is the area of the floor mat in square metres?

19. The gym teacher told the students to run twice around the perimeter of the school field. The area of the square field is 28 900 m^2. What distance did the students run?

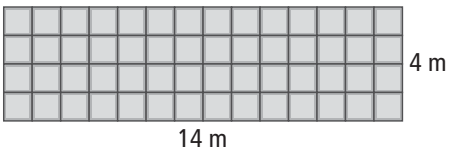
20. Adam's uncle has instructions for building a shed. One page of the instructions, shown below, is not very clear.



a) What is the area of the rectangle?

b) What is the side length of the square?

21. Kate is going to put a patio in her backyard. The patio stones she is using each have an area of 1 m^2. She has created the rectangular design shown.



a) What is the area of the patio?

b) What are the dimensions of another rectangular patio she could build with the same area?

c) Kate decides to make a patio with the same area but she wants it to be a square with whole number side lengths. Is this possible? Explain your reasoning.

22. The world's largest city square is Tiananmen Square in Beijing, China. It has an area of 396 900 m^2.

a) What are the dimensions of the square?

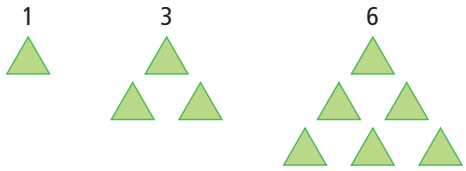
b) If the square had dimensions of 629 m by 629 m, what would be the area?

c) If the square had an area less than 394 000 m^2 and greater than 386 000 m^2, what are all of the possible whole number dimensions that it could have?

23. A helicopter landing pad has a square shape. The area is 400 m^2. Use prime factorization to find the side length of the pad.

Extend

24. The first three triangular numbers are



a) What are the next three triangular numbers?

b) Add together any two consecutive triangular numbers. What do you notice about the sums?

25. A square digital photo on the computer has an area of 144 cm^2.

a) What is the side length of the photo?

b) The photo is enlarged so that the side length is now 36 cm. What is the area of the enlarged photo?

c) How many times as large as the original area is the enlarged area?

d) How many times as large as the original side length is the enlarged side length?

e) Use what you know about the square root of a perfect square to identify the relationship between the numbers in parts c) and d).

Imagine your dog is 80 cm tall and your cat is 40 cm tall. How many times as tall as your cat is your dog? What operation did you perform?

26. **a)** Determine which of the following numbers are perfect squares: 10, 100, 1000, 10 000, 100 000.

b) State the square root of each perfect square.

c) Choose one of the numbers that is not a perfect square. Explain how you know that it is not a perfect square.

d) Describe a quick method for determining mentally if the numbers are perfect squares.

e) Use your method in part d) to decide if 1 000 000 000 is a perfect square.

27. **a)** Determine the square root of each number: 6400, 640 000, 64 000 000.

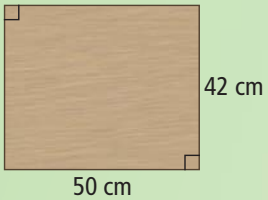
b) Describe a quick method for determining mentally the square root of each number in part a).

c) Explain why this method does not work for evaluating $\sqrt{640}$.

d) Use your method in part b) to evaluate $\sqrt{640\,000\,000\,000}$. Explain how you determined the answer.

MATH LINK

Chess is played on a square board. The board is made up of 32 white squares and 32 dark squares.



You decide to make your own chessboard. You are going to cut the board out of the 42 cm x 50 cm piece of wood shown.

Each square on the board will have whole number side lengths. The chess pieces fit on squares that are no smaller than 9 cm^2. What are all of the possible dimensions that your board could have?