10.2

Modelling and Solving Two-Step Equations: $ax + b = c$

Focus on...

After this lesson, you will be able to...

- model problems with two-step linear equations
- solve two-step linear equations and show how you worked out the answer

Cali borrowed $19 from her brother to purchase a CD. The next day, she paid back $3. She will pay back the rest at a rate of $4/week. Suggest ways that Cali might determine how long it will take to pay back her brother.

Explore the Math

How do you solve two-step equations of the form $ax + b = c$?

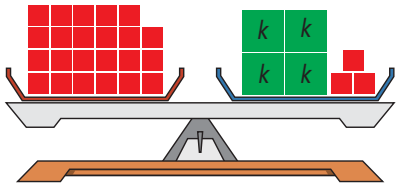
Example 1: Model With a Balance Scale

The city in Canada with the highest average wind speed is St. John's, Newfoundland. The city with the lowest average wind speed is Kelowna, British Columbia. The relationship between the wind speeds can be modelled using the equation $s = 4k + 3$, where s represents the wind speed in St. John's and k represents the wind speed in Kelowna. If the average wind speed in St. John's is 23 km/h, what is the average wind speed in Kelowna?

Solution

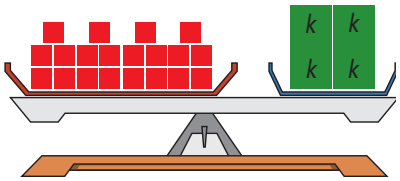
Substitute the known wind speed into the equation.
The wind speed for St. John's is 23 km/h.
$23 = 4k + 3$

You can model this equation using blocks and a scale.



To isolate the variable, first remove the three unit blocks from the right side of the scale. To keep the scale balanced, you must remove the same number of unit blocks from the left side of the scale.

There are four k blocks on the right side of the scale. There are 20 unit blocks on the left side of the scale. For the scale to balance, each k block must have a mass of five unit blocks.



The average wind speed in Kelowna is 5 km/h.

Check:
Left Side $= 23$

Right Side $= 4k + 3$
$= 4(5) + 3$
$= 20 + 3$
$= 23$

Left Side = Right Side
The solution is correct.

WWW Web Link

To practise solving linear equations using a balance scale, go to www.mathlinks8.ca and follow the links.

Show You Know

Solve each equation by drawing a diagram of a balance scale and blocks.

a) $6n + 6 = 12$ **b)** $13 = 9 + 2p$

Example 2: Model With Algebra Tiles

A cow sleeps 7 h a day. This amount of sleep is 1 h less than twice the amount an elephant sleeps a day. How long does an elephant sleep?

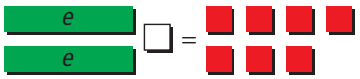
Solution

Let e represent the hours an elephant sleeps.

A cow sleeps 1 h less than twice what an elephant sleeps, or $2e - 1$.

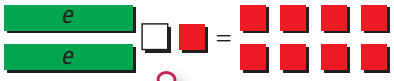
A cow sleeps 7 h.

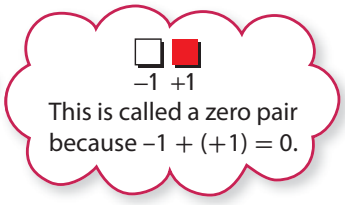
$2e - 1 = 7$



One less means you need to subtract 1, and *twice* means you need to multiply by 2.

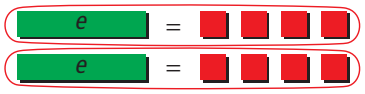
To isolate the variable, first add one positive 1-tile to both sides.





The negative 1-tile and positive 1-tile on the left side equal zero.

The two variable tiles must have the same value as the eight positive 1-tiles on the right side of the model. Each variable tile must then have a value of four positive 1-tiles.



An elephant sleeps 4 h a day.

Check:

Left Side $= 2e - 1$ Right Side $= 7$

$= 2(4) - 1$

$= 8 - 1$

$= 7$

Left Side = Right Side

The solution is correct.

Literacy Link

To solve a problem, you sometimes need to translate words into an equation. For example, *two more* means you need to add 2, and *three times* means you need to multiply by 3. What other words translate into math operations?

Literacy Link

Order of Operations

When substituting a value into the equation, make sure to use the correct order of operations:

- first, multiply and divide in order from left to right
- finally, add and subtract in order from left to right

Show You Know

Model each equation with algebra tiles. Then, solve.

a) $2g + 4 = -6$ **b)** $-2r - 7 = -11$

How do you represent $-2r$ using algebra tiles?

Example 3: Apply the Opposite Operations

Cali borrowed $19 from her brother. The next day, she paid back $3. To pay off the rest of the debt, she will give him $4/week. How many weeks will it take her to pay off the debt?

Solution

Let w represent the number of weeks.

Cali is paying off $4/week and has already paid $3. The total she will pay is $4w + 3$. She owes a total of $19.

$4w + 3 = 19$

The amount Cali still needs to pay back is $4 times the number of weeks, or *"4w"*. The amount of $3 that she has already paid back is represented by "+ 3."

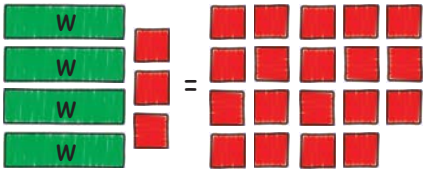
If you think of money owed as being negative, you can use the equation $-4w - 3 = -19$. When you solve it, the value of w is still the same.

Isolate the variable w to solve the equation.

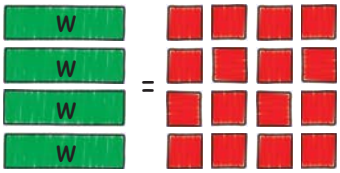
$$\begin{aligned} 4w + 3 &= 19 \\ 4w + 3 - 3 &= 19 - 3 \quad \text{Subtract 3 from both sides of the equation.} \\ 4w &= 16 \\ \frac{4w}{4} &= \frac{16}{4} \quad \text{Divide both sides of the equation by 4.} \\ w &= 4 \end{aligned}$$

It will take Cali four weeks to pay off her debt.

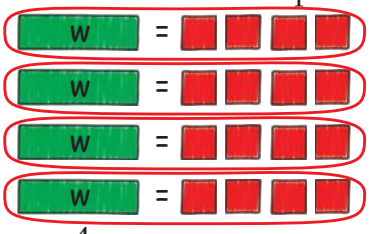
Check:



Subtract three positive 1-tiles from both sides.



The four variable tiles must have the same value as the 16 positive 1-tiles on the right side of the model. Each variable tile must then have a value of four positive 1-tiles.



$w = 4$

The solution is correct.

Literacy Link

Reverse Order of Operations

When isolating a variable, follow the reverse order of operations:

- add and/or subtract
- multiply and/or divide

Strategies

Draw a Diagram

Show You Know

Solve by applying the opposite operations.

a) $4 + 26g = -48$ **b)** $-3x + 7 = 19$

Key Ideas

- To solve an equation, isolate the variable on one side of the equal sign. When undoing the operations performed on the variable, follow the reverse order of operations:
  - add and/or subtract
  - multiply and/or divide

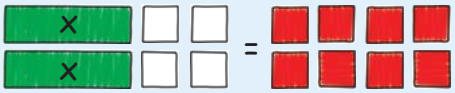
$$\begin{aligned} 2x - 4 &= 8 \\ 2x - 4 + 4 &= 8 + 4 \\ 2x &= 12 \\ \frac{2x}{2} &= \frac{12}{2} \\ x &= 6 \end{aligned}$$

- Two methods you can use to check your solution are substitution and drawing a diagram:
  - Substitute your answer into the equation. Both sides should have the same value.

Left Side $= 2x - 4$ Right Side $= 8$
$= 2(6) - 4$
$= 12 - 4$
$= 8$
Left Side = Right Side

The solution is correct.

  - Draw a diagram to model the equation.



Add four positive 1-tiles to both sides.

The four negative 1-tiles and the four positive 1-tiles on the left side equal zero. The two variable tiles must have the same value as the 12 positive 1-tiles. That means each variable tile must have a value of six positive 1-tiles .

The solution of $x = 6$ is correct.

Communicate the Ideas

1. Draw diagrams to show how you would solve the equation $24 = 14 - 5x$ using algebra tiles. Explain each step in words.

2. a) Describe how you would isolate the variable in the equation $5x + 10 = 40$.

b) If the equation is changed to $5x - 10 = 40$, would you use the same process to isolate the variable? Explain.

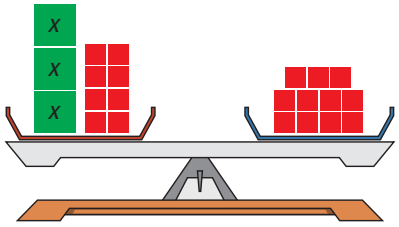
Check Your Understanding

Practise

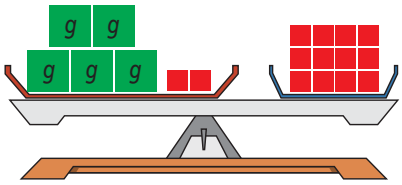
For help with #3 and #4, refer to Example 1 on page 380–381.

3. Solve the equation modelled by each balance scale. Check your solution.

a)

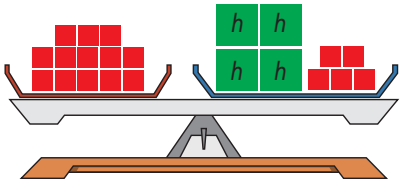


b)

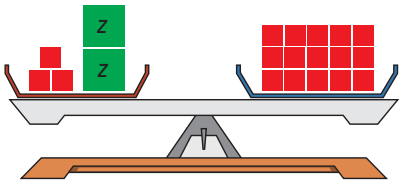


4. Solve the equation represented by each balance scale. Verify your solution.

a)



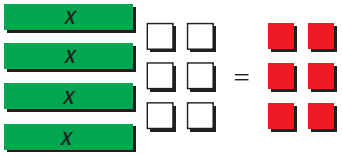
b)



For help with #5 and #6, refer to Example 2 on page 382.

5. Solve each equation modelled by the algebra tiles. Check your solution.

a)



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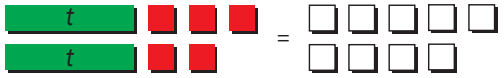
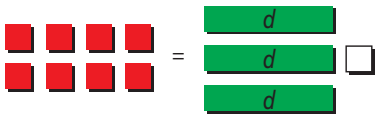
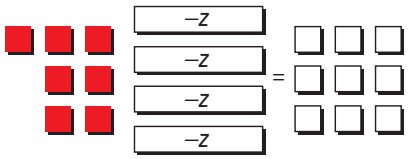


6. Solve each equation represented by the algebra tiles. Verify your solution.

a)



b)



For help with #7 to #10, refer to Example 3 on page 383.

7. What is the first operation you should perform to solve each equation?

a) $4r - 2 = 14$

b) $3 - 3x = -9$

c) $-22 = -10 + 2m$

d) $53 = -9k - 1$

8. What is the second operation you should perform to solve each equation in #7?

9. Solve each equation. Check your answer.

a) $6r + 6 = 18$

b) $4m + 8 = 12$

c) $39 + 9g = 75$

d) $-37 = 8f - 139$

10. Solve. Verify your answer.

a) $-17 = 3k + 4$

b) $29 = -14n + 1$

c) $8x - 7 = -31$

d) $-10 = 4n - 12$

Apply

11. Show whether $x = -3$ is the solution to each equation.

a) $-8x - 1 = 25$

b) $3 - 7x = -24$

c) $29 = -10x - 1$

d) $30 = 6x + 12$

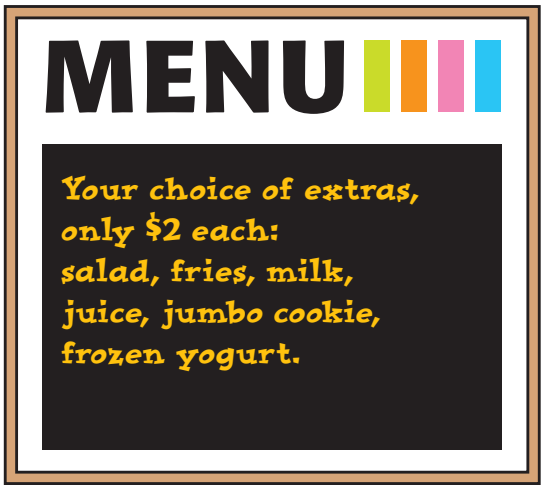
12. Matt is saving \$750 to buy a clothes dryer. If he triples the amount he has saved so far, he will have \$30 more than he needs. The situation can be modelled as $3s - 30 = 750$, where s represents the amount he has saved so far.

a) Explain how $3s - 30 = 750$ models the situation.

b) How much money has Matt saved so far?

c) What other strategy could you use to determine Matt's savings?

13. You are buying lunch at Sandwich Express. The cost is \$4 for a sandwich and \$2 each for your choice of extras. You have \$10. The equation to determine how many extras you can get is $10 = 2e + 4$, where e is the number of extras. How many extras can you buy if you spend all of your money?



14. The percent of elementary school students who choose hockey as their favourite physical activity is 14%. This percent of students is 2% more than four times the percent who choose skiing.

a) Let s represent the percent of students who choose skiing. What equation models this situation?

b) Solve the equation to find the percent of students who choose skiing.

15. If Jennifer doubled the money that she has in her account now and then took out \$50, she would have enough left in her account to buy a new bike that costs \$299. Write and solve an equation to determine how much money Jennifer has now.

16. A classroom's length is 3 m less than two times its width. The classroom has a length of 9 m. Write and solve an equation to determine the width of the classroom.

17. An eagle is hunting a bird in flight. The eagle begins its descent from a height of 74 m. The eagle reaches its prey at a height of 3 m. This situation can be modelled using the formula $74 = 3 + 6t$, where t represents the time in seconds.

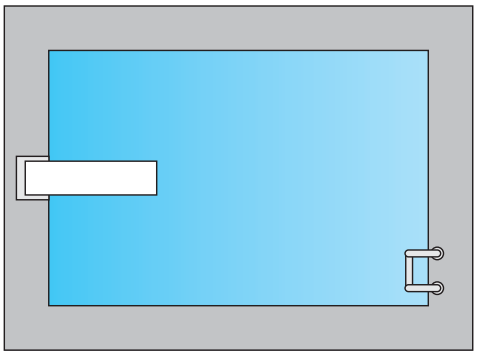
a) What do you think the value of 6 represents in the equation?

b) After how many seconds does the eagle reach its prey? Give your answer to the nearest tenth of a second.

Extend

18. The base of an isosceles triangle is 6 m less than two times one side. The base is 24 m. What is the area of the triangle?

19. The deck around a swimming pool has the same width all the way around. The perimeter of the pool is 50 m. The outside perimeter of the deck is 74 m. What is the width of the deck?



20. The variable m is a positive integer. The variable n is an integer from 0 to 9. Identify all of the values for m that would satisfy the equation $3m + n = 2008$.

21. Mallika walked at 2 km/h for 2 h and then cycled at x km/h for 3 h. If the average speed for the whole journey was 3 km/h, how fast did she cycle? Give your answer to the nearest tenth of a kilometre per hour.

MATH LINK

When any object falls, it picks up more and more speed as it falls. In fact, a falling object increases its speed by about 10 m/s for every second it falls.

Suppose a stone is dislodged from the side of a canyon and falls with an initial speed of 5 m/s. It hits the water below it at a speed of 45 m/s.

Write and solve an equation to determine the amount of time the stone fell before it hit the water.