10.3

Modelling and Solving Two-Step Equations: $\frac{x}{a} + b = c$

Focus on...

After this lesson, you will be able to...

- model problems with two-step linear equations
- solve two-step linear equations and show how you worked out the answer

The mass of a Persian cat is typically 2 kg less than $\frac{1}{3}$ of the average mass of a border collie. The average mass of a Persian cat is 4 kg. Describe how you might determine the average mass of a border collie.

Explore the Math

Materials

- algebra tiles

How do you model and solve two-step equations of the form $\frac{x}{a} + b = c$?

1. Use d to represent the average mass of a border collie. What is an equation that models the relationship between the masses of the border collie and the Persian cat?

2. How could you use a model or diagram to represent your equation?

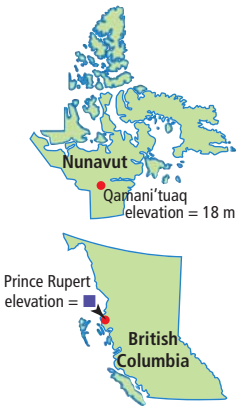
3. Use your model or diagram to help you solve this equation.
   a) What is the first thing you do to isolate d?
   b) What equation does your model or diagram represent now?
   c) What do you do next?
   d) What is the average mass of a border collie?

Reflect on Your Findings

4. a) Why is this type of equation called a two-step equation?
   b) How is solving an equation of the form $\frac{x}{a} + b = c$ similar to solving one of the form $ax + b = c$? How is it different?

Example 1: Model Equations

The elevation of Qamani'tuaq, Nunavut, is 1 m less than $\frac{1}{2}$ the elevation of Prince Rupert, British Columbia. If the elevation of Qamani'tuaq is 18 m, what is the elevation of Prince Rupert?



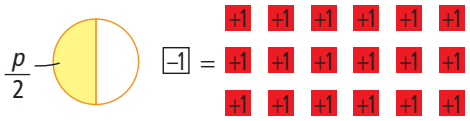
Did You Know?

The community of Qamani'tuaq, Nunavut, is also known as Baker Lake.

Solution

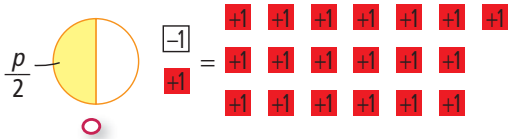
Let p represent the elevation of Prince Rupert.

The equation that models this situation is $\frac{p}{2} - 1 = 18$.



What part of the equation represents $\frac{1}{2}$ the elevation? What part of the equation represents 1 m less?

To isolate the variable, first add one red +1 square to both si des.



You need two equal parts to fill the circle.

The $\frac{1}{2}$ circle must have the same value as +19.
Multiply by 2 to fill the circle.
To balance the equation, multiply +19 by 2.
The variable p must then have a value of $2 \times 19 = 38$.
The elevation of Prince Rupert is 38 m.

Check:

Left Side $= \frac{p}{2} - 1$ Right Side $= 18$

$= \frac{38}{2} - 1$

$= 19 - 1$

$= 18$

Left Side = Right Side

The solution is correct.

Show You Know

Solve by modelling each equation.

a) $\frac{x}{4} - 5 = -7$ **b)** $\frac{-p}{3} + 1 = -4$

Example 2: Apply the Reverse Order of Operations

During the 2006–2007 NHL season, Kristian Huselius of the Calgary Flames had a total of 41 more than $\frac{1}{2}$ the number of shots on goal as Jarome Iginla. If Huselius had 173 shots on goal, how many did Iginla have?

Use the reverse order of operations. Add and subtract first, then multiply and divide.

Solution

Let j represent the number of shots on goal Jarome Iginla had.

This situation can be modelled with the equation $\frac{j}{2} + 41 = 173$.

$\frac{j}{2} + 41 - 41 = 173 - 41$ Subtract 41 from both sides of the equation.

$\frac{j}{2} = 132$

$\frac{j}{2} \times 2 = 132 \times 2$ Multiply both sides of the equation by 2.

$j = 264$

Why would you not use models to solve this problem?

Jarome Iginla had 264 shots on goal during the 2006–2007 season.

Check:

Left Side $= \frac{j}{2} + 41$ Right Side $= 173$

$= \frac{264}{2} + 41$

$= 132 + 41$

$= 173$

Left Side = Right Side

The solution is correct.

Show You Know

Solve by applying the reverse order of operations.

a) $\frac{-x}{12} - 6 = 4$ **b)** $-4 = 3 + \frac{k}{7}$

Key Ideas

- To solve an equation, isolate the variable on one side of the equal sign. When undoing the operations performed on the variable, follow the reverse order of operations:
  - subtract and/or add
  - multiply and/or divide

$$\frac{x}{-4} + 3 = 5$$
$$\frac{x}{-4} + 3 - 3 = 5 - 3$$
$$\frac{x}{-4} = 2$$
$$\frac{x}{-4} \times (-4) = 2 \times (-4)$$
$$x = -8$$

$$5 = 2 - \frac{n}{4}$$
$$5 - 2 = 2 - 2 - \frac{n}{4}$$
$$3 = -\frac{n}{4}$$
$$3 \times 4 = -\frac{n}{4} \times 4$$
$$12 = -n$$
$$12 \div (-1) = -n \div (-1)$$
$$-12 = n$$

- One method you can use to check your answer is substituting it back into the equation. Both sides of the equation should have the same value.

Left Side $= \frac{x}{-4} + 3$ Right Side $= 5$

$= \frac{-8}{-4} + 3$

$= 2 + 3$

$= 5$

Left Side = Right Side

The solution is correct.

Communicate the Ideas

1. Describe a situation that can be modelled with the equation $\frac{x}{4} - 2 = 3$.

2. Describe how to isolate the variable when solving $12 - \frac{n}{5} = 6$. Compare your answer with a classmate's.

3. Manjit believes that the first step in solving the equation $\frac{x}{-4} + 7 = 9$ is to multiply both sides of the equation by -4 as shown.

$\frac{x}{-4} \times (-4) + 7 = 9 \times (-4)$

Is he correct? Explain.

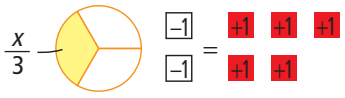
Check Your Understanding

Practise

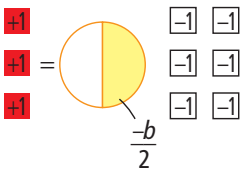
For help with #4 to #7, refer to Example 1 on page 389.

4. Solve the equation modelled by each diagram. Check your solution.

a)

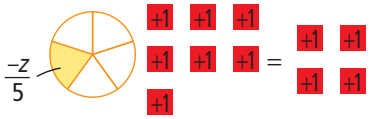


b)

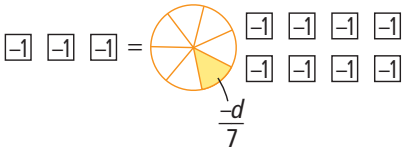


5. Solve the equation represented by each diagram. Verify your solution.

a)



b)



6. Draw a model for each equation. Then, solve. Verify your answer.

a) $-5 + \frac{g}{-2} = 3$ **b)** $-3 = 7 + \frac{n}{5}$

7. For each equation, draw a model. Then, solve. Check your answer.

a) $\frac{f}{-5} + 3 = -2$ **b)** $-1 = \frac{n}{8} - 4$

For help with #8 to #11, refer to Example 2 on page 390.

8. What is the first operation you should perform to solve each equation?

a) $\frac{t}{-5} + 12 = 9$ **b)** $\frac{p}{13} - 2 = -3$

c) $\frac{-k}{12} + 6 = 15$ **d)** $14 = 11 - \frac{x}{3}$

9. What is the second operation you should perform to solve each equation in #8?

10. Solve each equation. Verify your answer.

a) $2 + \frac{m}{3} = 18$ **b)** $\frac{c}{-8} - 8 = -12$

c) $16 = 9 + \frac{b}{-8}$ **d)** $-3 = \frac{n}{-7} + 19$

11. Solve. Check your answer.

a) $4 + \frac{j}{-8} = 8$ **b)** $\frac{r}{2} - 12 = -12$

c) $15 = -5 + \frac{x}{-6}$ **d)** $-2 = \frac{n}{13} - 17$

Apply

12. Show whether $n = -72$ is the solution to each equation.

a) $6 + \frac{n}{9} = 14$ **b)** $2 = 14 + \frac{n}{6}$

c) $\frac{n}{-3} + 6 = -18$ **d)** $-17 = \frac{n}{36} - 15$

13. The amount of sleep needed each night by people 18 years old or younger can be modelled by the equation $s = 12 - \frac{a}{4}$, where the amount of sleep in hours is s, and the age in years is a.

a) If 10 h is the amount of sleep Brian needs, how old is he likely to be?

b) Natasha is 13. She gets 8 h of sleep each night. Is this enough? Explain your reasoning.

14. The cost of a concert ticket for a student is \$2 less than one half of the cost for an adult. The cost of the student ticket is \$5. Let a represent the cost of an adult ticket. Write and solve an equation to determine the cost of an adult ticket.

15. In the following formula, T is the air temperature in degrees Celsius at an altitude of h metres, and t is the ground temperature in degrees Celsius:
$T = t - \frac{h}{150}$.

a) If the ground temperature is 25 °C, what is the temperature outside an aircraft at an altitude of 7500 m?

b) What is the altitude of the same plane if the outside air temperature is −35 °C?

16. In Canada, the percent of secondary school students who say their favourite subject is science is 1% less than $\frac{1}{2}$ of the number of students who choose math. The percent of students who prefer science is 6%. Write and solve an equation to determine what percent of students prefer math.

Extend

17. The recommended energy requirement per day for 14-year-old boys depends on how active they are. The requirement can be modelled by the following equations, where a is the age and C is the number of Calories.

Active	Moderately Active
$a = \frac{C}{100} - 17$	$a = \frac{C}{100} - 13$

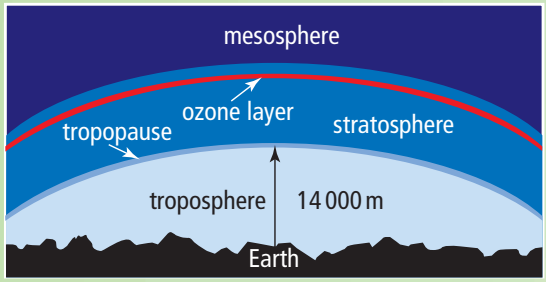
a) Tom is an active 14-year-old. What is the recommended number of Calories he should consume?

b) Juan is a moderately active 14-year-old boy. If he consumes 2831 Calories per day, is this greater or fewer Calories than the recommended amount?

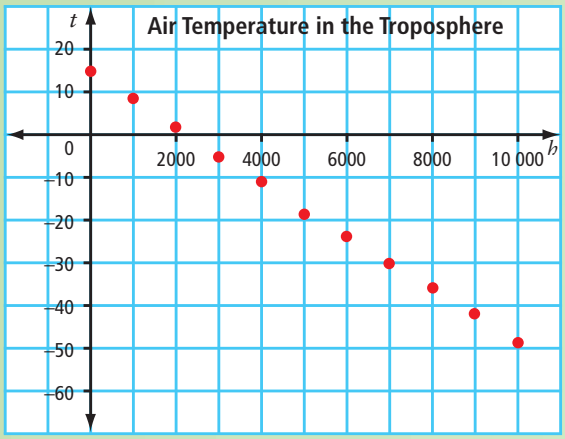
c) The recommended requirement for a moderately active 14-year-old girl is 2100 Calories. Model this energy requirement by determining the value for x in the equation $a = \frac{C}{100} - x$.

MATH LINK

Meteorologists rely on models of our atmosphere to help them understand temperature and pressure differences, humidity, and a wide range of other variables. An important part of our atmosphere is the troposphere. It is the lowest layer of the atmosphere, where humans live and where weather occurs.



The equation that models air temperature change in the troposphere is $t = 15 - \frac{h}{154}$, where t is the temperature, in degrees Celsius, and h is the altitude, in metres.



a) What patterns do you see in the graph?

b) What connections do you see between the graph and the equation?

c) At what height in the troposphere is the temperature 0 °C?