Modelling and Solving Two-Step Equations: $a(x+b)=c$

Kia plans to make a square Star Quilt for her grandmother. The quilt will have a 4-cm wide border around it. Kia wants the perimeter of the completed quilt to be 600 cm. How can Kia decide how long each side of the quilt should be before she adds the border?

Explore the Math

Materials

• algebra tiles

How do you solve equations of the form $a(x+b)=c$?

Viktor missed yesterday's math class. Jackie will show him how to model and solve the equation $3(x-5)=-6$

1. **a)** Use a variable tile to represent x.

   b) How will you use negative 1-tiles to represent -5?

2. **a)** How many sets of $x-5$ will you include in your model? Explain.

   b) How will you complete your model of the equation?

3. **a)** What is the first thing you do to isolate the variable tile?

   b) What equation does your model represent now?

   c) What do you need to do to solve the equation?

4. What is the unknown value of x?

Reflect on Your Findings

5. What steps did you take to solve the equation?

Did You Know?

The centre of a Star Quilt is in the shape of the traditional eight-pointed morning star of the Lakota and Dakota Sioux. The Star Quilt is a symbol of tradition to the Plains peoples.

Example 1: Model With Algebra Tiles

A flower garden is in the shape of a rectangle. The length of the garden is 2 m longer than the length of the shed beside it. The width of the garden is 4 m. If the area of the garden is 20 m², what is the length of the shed?
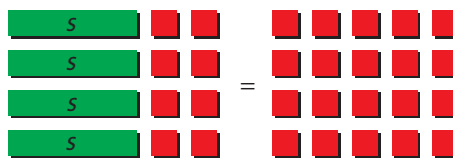
4 m

s | 2 m

$A = 20\ m^2$
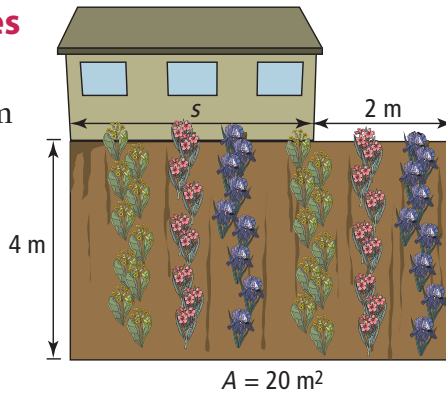
Solution

Let s represent the unknown length of the shed. The length of the garden can be represented by $s + 2$. The width of the garden is 4 m. The equation that models the area of the garden is $4(s + 2) = 20$.
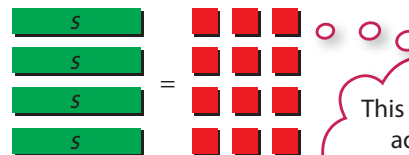
What is the formula for the area of a rectangle?

There are four groups of $(s + 2)$. That means there are four variable tiles and eight positive 1-tiles on the left side of the equation.

To isolate the variable, subtract eight positive 1-tiles from both sides of the equal sign.

This is the same as adding eight negative 1-tiles to both sides of the equal sign.

There are now four variable tiles on the left side and 12 positive 1-tiles on the right side.

The four variable tiles must have the same value as the 12 positive 1-tiles.

Each variable tile must then have a value of three positive 1-tiles.
The length of the shed is 3 m.

Check:
Left Side $= 4(s + 2)$ Right Side $= 20$
$\qquad = 4(3 + 2)$
$\qquad = 4(5)$
$\qquad = 20$
$\qquad$ Left Side $=$ Right Side
The solution is correct.

Show You Know

Solve by modelling the equation.
a) $2(g + 4) = -8$ **b)** $3(r - 2) = 3$

Literacy Link

When substituting a value into an equation, be sure to use the correct order of operations:
- Brackets.
- Multiply and divide in order from left to right.
- Add and subtract in order from left to right.

Example 2: Solve Equations

Kia is making a square quilt with a 4-cm wide border around it. She wants the completed quilt to have a perimeter of 600 cm. What must the dimensions of Kia's quilt be before she adds the border?
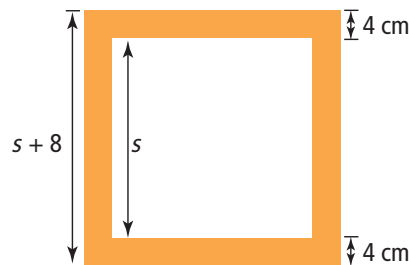
Solution

Let *s* represent the unknown side length of the quilt before the border is added. A border of 4 cm is added to each side. That means the side length of the quilt after the border is added is $s + 8$. Model with the equation $4(s + 8) = 600$.

> The length must be multiplied by 4 because there are four sides to the square quilt.

Method 1: Divide First

Isolate the variable *s*.

$$4(s + 8) = 600$$
$$\frac{4(s + 8)}{4} = \frac{600}{4} \qquad \text{Divide by 4 to undo the multiplication.}$$
$$s + 8 = 150$$
$$s + 8 - 8 = 150 - 8 \qquad \text{Subtract 8 to undo the addition.}$$
$$s = 142$$

The quilt dimensions before adding the border should be 142 cm × 142 cm.

Method 2: Use the Distributive Property First

Isolate the variable *s*.

$$4(s + 8) = 600$$
$$4s + 32 = 600 \qquad \text{Multiply both } s \text{ and } 8 \text{ by 4.}$$
$$4s + 32 - 32 = 600 - 32 \qquad \text{Subtract 32 from both sides of the equation.}$$
$$4s = 568$$
$$\frac{4s}{4} = \frac{568}{4} \qquad \text{Divide both sides of the equation by 4.}$$
$$s = 142$$

The quilt dimensions before adding the border should be 142 cm × 142 cm.

> **Literacy Link**
>
> The *distributive property* states that $a(b + c)$ equals $a \times b + a \times c$.

> To use the distributive property, multiply the terms in the brackets by 4.
>
> $4(s + 8) = 4 \times (s + 8)$
> $\quad = (4 \times s) + (4 \times 8)$
> $\quad = 4s + 32$

Check:

Left Side $= 4(s + 8)$ Right Side $= 600$
$\qquad\quad = 4(142 + 8)$
$\qquad\quad = 4(150)$
$\qquad\quad = 600$

Left Side $=$ Right Side

The solution is correct.

Show You Know

Solve each equation.

a) $-2(x - 3) = 12$ **b)** $-20 = 5(3 + p)$

- To solve an equation, isolate the variable on one side of the equal sign.
- When undoing the operations performed on the variable, use opposite operations.
- Solve an equation of the form $a(x + b) = c$ by dividing first or by using the distributive property first.

  Divide First:

  $-4(x - 7) = 16$

  $\dfrac{-4(x - 7)}{-4} = \dfrac{16}{-4}$ Divide by -4 to undo the multiplication.

  $x - 7 = -4$

  $x - 7 + 7 = -4 + 7$ Add 7 to undo the subtraction.

  $x = 3$

  Use the Distributive Property First:

  $-4(x - 7) = 16$

  $-4x + 28 = 16$ Use the distributive property.

  $-4x + 28 - 28 = 16 - 28$ Subtract 28 to undo the addition.

  $-4x = -12$

  $\dfrac{-4x}{-4} = \dfrac{-12}{-4}$ Divide by -4 to undo the multiplication.

  $x = 3$

- One method you can use to check your answer is substituting it back into the equation. Both sides of the equation should have the same value.

  Left Side $= -4(x - 7)$ Right Side $= 16$

  $= -4(3 - 7)$

  $= -4(-4)$

  $= 16$

  Left Side $=$ Right Side

  The solution is correct.

Communicate the Ideas

1. Draw diagrams to show how you would solve the equation $4 = 2(v - 3)$ using algebra tiles. Explain each step in words.

2. Julia and Chris are solving the equation $-18 = -6(x + 2)$. Is either strategy correct? Explain.

Julia: First, I subtract 2 from both sides. Then, I divide both sides by –6.

Chris: I start by dividing $-6(x + 2)$ by –6. Then, I subtract 2 from both sides.

3. Describe a situation that can be modelled with the equation $2(r + 3) = -6$.

Check Your Understanding

Practise

For help with #4 to #7, refer to Example 1 on page 395.

4. Solve the equation modelled by each diagram. Check your solution.

a)

b)

5. Solve the equation represented by each diagram. Verify your answer.

a)

b)

6. Model and then solve each equation. Check your answer.

a) $3(t - 2) = 12$ b) $6(j - 1) = -6$

7. Model and then solve each equation. Verify your solution.

a) $2(3 + p) = 8$ b) $0 = 7(n - 2)$

For help with #8 and #9, refer to Example 2 on page 396.

8. Solve each equation. Check your answer.

a) $6(r + 6) = -18$

b) $4(m - 3) = 12$

c) $3(1 + g) = -75$

d) $36 = 6(f + 13)$

9. Solve. Verify your solution.

a) $-21 = 3(k + 3)$

b) $42 = -14(n - 11)$

c) $8(x - 7) = -32$

d) $-10 = -5(w + 13)$

Apply

10. Show whether $x = -4$ is the solution to each equation.

a) $-8(x - 1) = 24$

b) $3(-8 - x) = -24$

c) $25 = -5(x - 1)$

d) $66 = 6(x + 7)$

11. The fence around Gisel's new tree is in the shape of an equilateral triangle. Gisel wants to increase the length of each side by 7 cm. The perimeter of her new fence will be 183 cm.

a) Let s represent the side length of the old fence. What equation models this situation?

b) Determine the length of each side of the old fence.

12. The amount of food energy per day required by hikers is modelled by the equation $e = -125(t - 122)$, where e is the amount of food energy, in kilojoules (kJ), and t is the outside temperature, in degrees Celsius.

a) If the outside temperature is -20 °C, how much food energy is required per day?

b) If a hiker consumes 19 000 kJ of food energy based on the outside temperature, what is the temperature?

13. Barney wants to frame a square picture of his dog. The frame he bought fits a picture with a perimeter no greater than 96 cm. He plans to put a 2-cm blue border around the picture.

   a) What equation models this situation?

   b) Determine the maximum dimensions that the picture can have.

Extend

14. A computer rental company charges by the hour: $5 for the first hour and $4 for every hour after that. The fee rate can be modelled with the equation $4(n - 1) = T - 5$, where n is a number of hours greater than zero and T is the rental fee, in dollars. Candy's rental fee was $17. For how many hours did she rent the computer?

15. A parking lot charges by the hour: $2 for the first hour and $3 for every hour after that. The formula used to calculate the number of hours someone has parked is $3(h - 1) = T - 2$, where h represents a number of hours greater than zero and T represents the total amount of the parking fee, in dollars. If Mark's parking fee is $8, how long did he park in the lot?

16. The distance between Andrew's house and his grandfather's apartment is 42 km.

   a) If Andrew rides his bike 2 km/h faster than his current speed, he could get there in 3 h. What is Andrew's current speed?

   b) If Andrew wants to get there in 2 h, how much faster than his current speed should he ride his bike?

   c) Do you think Andrew can get there in 2 h? Explain.

MATH LINK

Some jobs require working the night shift, such as from midnight to 8:30 a.m. Other jobs require working in isolated areas or under hazardous conditions. Depending on the job, the wage may be increased by a certain amount per hour or per month. This increase is called a premium.

a) Research and describe three different jobs that pay hourly or monthly wages plus a premium.

b) For each job, model the pay using an equation.