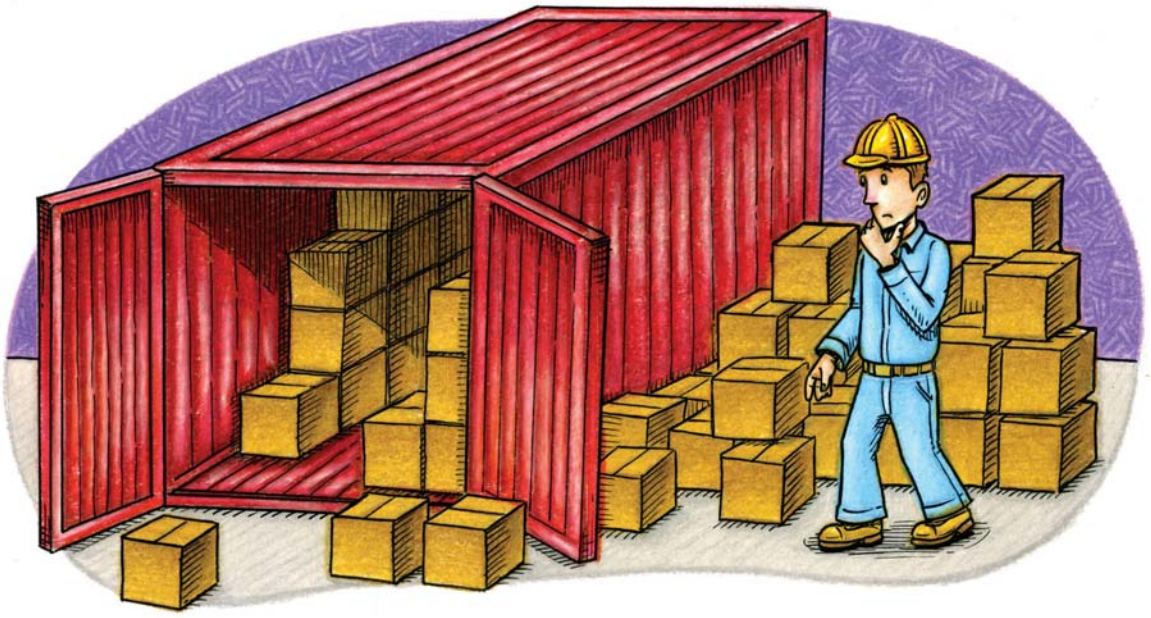
7.1 Understanding Volume

Focus on...

After this lesson, you will be able to...

- explain the meaning of volume
- determine the volume of a right rectangular prism, right triangular prism, and right cylinder
- show that orientation does not affect volume



Bruce has just taken on a part-time job at a local shipping company. He is packing boxes into a shipping container. He knows how many boxes he can fit on the bottom of the container. How can he use this information to figure out how many boxes the shipping container will hold?

Materials

- centimetre cubes

base (of a prism or cylinder)

- any face of a prism that shows the named shape of the prism
- the base of a rectangular prism is any face
- the base of a triangular prism is a triangular face.
- the base of a cylinder is a circular face

height (of a prism or cylinder)

- the perpendicular distance between the two bases of a prism or cylinder

Explore the Math

How does the area of the base of a right prism relate to its volume?

1. **a)** Use centimetre cubes to build models of four different right rectangular prisms.
   b) What is the area of the **base** for each model? Record your data.
   c) What is the **height** of each model? Record your data.

2. How does the number of cubes help to determine the **volume** of each rectangular prism? What is the volume of each prism? Record your data.

One centimetre cube is equal to 1 cm^3.

volume

- the amount of space an object occupies
- measured in cubic units

Reflect on Your Findings

4. a) What is the relationship between the area of the base, the height of the prism, and the volume of a rectangular prism?

b) Do you think this same relationship exists for the volume of a right triangular prism? Explain your reasoning.

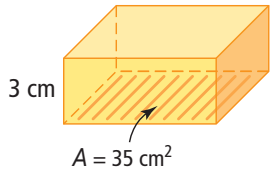
Literacy Link

Read 1 cm^3 as "one cubic centimetre."

Example 1: Determine the Volume Using the Base and the Height

Determine the volume of each right prism or cylinder.

a)



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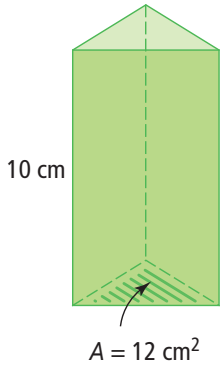

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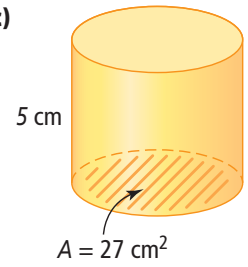


Literacy Link

Prisms and cylinders in this chapter are *right* prisms and *right* cylinders.

Solution

a) The prism is a right rectangular prism.
The area of the rectangular base is 35 cm^2.
The height of the prism is 3 cm.
Volume = area of base × height of prism
$V = 35 \times 3$
$V = 105$
The volume of the right rectangular prism is 105 cm^3.

Why are the units for volume in cm^3?

b) The prism is a right triangular prism.
The area of the triangular base is 12 cm^2.
The height of the prism is 10 cm.
Volume = area of base × height of prism
$V = 12 \times 10$
$V = 120$
The volume of the right triangular prism is 120 cm^3.

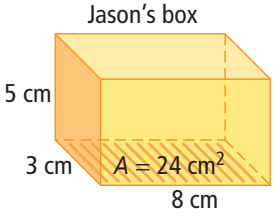
c) The cylinder is a right cylinder.
The area of the circular base is 27 cm^2.
The height of the cylinder is 5 cm.
Volume = area of base × height of cylinder
$V = 27 \times 5$
$V = 135$
The volume of the right cylinder is 135 cm^3.

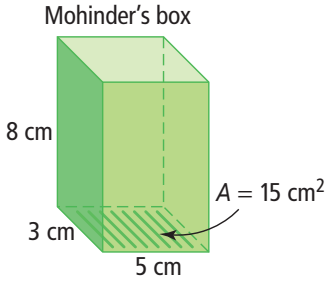
Show You Know

What is the volume of the right cylinder?

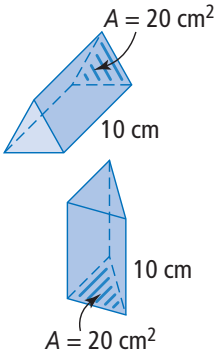
Example 2: Determine the Volume Using Different Orientations

Jason and Mohinder have two boxes with the same dimensions, 5 cm × 3 cm × 8 cm. Jason's box is short, with a height of 5 cm. Mohinder's box is taller; its height is 8 cm. Mohinder says his box has a larger volume than Jason's box. Is he correct?





orientation
- the different position of an object formed by translating, rotating, or reflecting the object



Solution

Determine the volume of each rectangular prism.

Jason's box: Base area of 24 cm^2
Volume = area of base × height
$V = 24 \times 5$
$V = 120$
The volume of the rectangular prism is 120 cm^3.

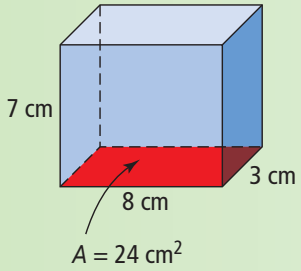
Mohinder's box: Base area of 15 cm^2
Volume = area of base × height
$V = 15 \times 8$
$V = 120$
The volume of the rectangular prism is 120 cm^3.

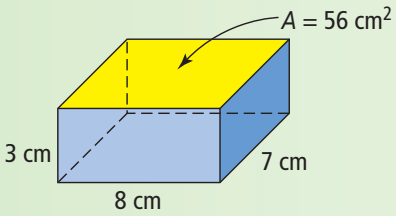
Mohinder is not correct. Both boxes have the same volume.

Do you think changing the **orientation** of a 3-D object ever affects the volume?

Show You Know

Which box has the greater volume? Explain your reasoning.

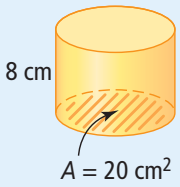




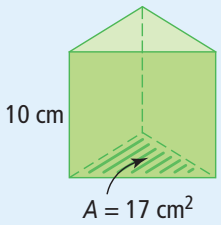
Key Ideas

- The volume of a right cylinder or a right prism can be determined by multiplying the area of the base by the height of the cylinder or prism.

Volume = area of base × height of cylinder
$V = 20 \times 8$
$V = 160$
The volume of the cylinder is 160 cm³.

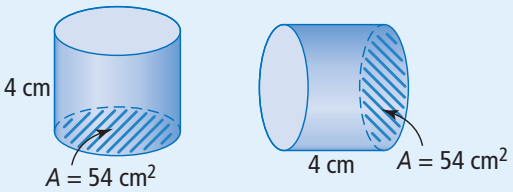


Volume = area of base × height of prism
$V = 17 \times 10$
$V = 170$
The volume of the triangular prism is 170 cm³.



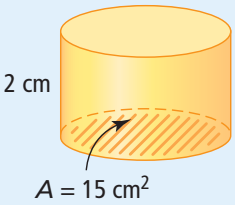
- Changing the orientation of a 3-D object does not affect its volume.

Volume = area of base × height
$V = 54 \times 4$
$V = 216$
The volume of the cylinder is 216 cm³.

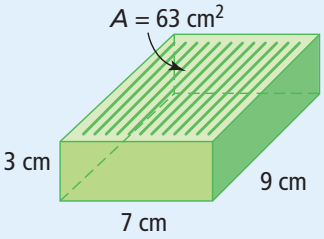


Communicate the Ideas

1. Evan calculated the volume of a right cylinder. Charlotte calculated the volume of a right rectangular prism. Did either of them make an error in their solutions? Explain how you know.



Volume = area of base X height
V = 15 X 2
V = 30
The volume of the cylinder is 30 cm³.



Volume = area of base X height
V = 63 X 7
V = 441
The volume of the rectangular prism is 441 cm³.

2. Does the volume of a right prism depend on which face is used as the base in the calculations? Use examples to support your position.

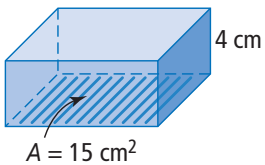
Check Your Understanding

Practise

For help with #3 and #4, refer to Example 1 on pages 247–248.

3. Determine the volume of each right prism or cylinder.

a)



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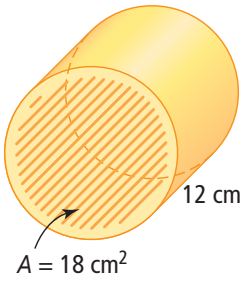

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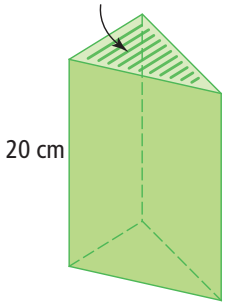


4. What is the volume of each right prism?

a) area of base = 12 cm², height = 8 cm

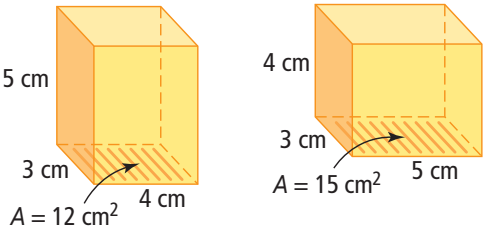
b) area of base = 18 cm², height = 4 cm

c) height = 9 cm, area of base = 14 cm²

For help with #5 and #6, refer to Example 2 on page 248.

5. Determine the volume of each right rectangular prism.

a)



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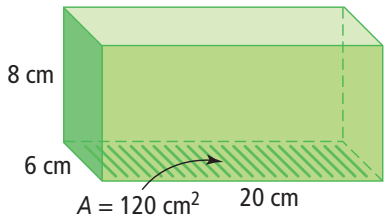
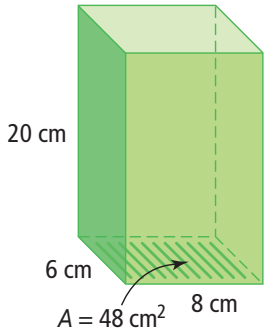
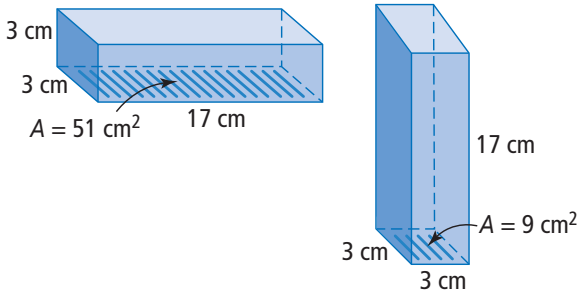




6. What is the volume of each right rectangular prism?

a)



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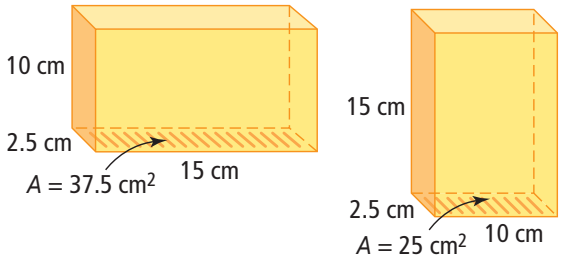


Apply

7. What is the height of each of the following right rectangular prisms?

a) volume = 32 cm^3, area of base = 8 cm^2

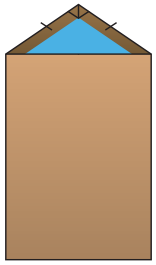
b) volume = 35 cm^3, area of base = 5 cm^2

c) area of base = 9 cm^2, volume = 36 cm^3

8. Nina uses 15 centimetre cubes to make the base of a rectangular prism. Determine the volume if the prism has a total of 5 layers of cubes. Show your thinking.

9. How many ways can you build a rectangular prism from 16 centimetre cubes? Use diagrams or centimetre cubes to show your designs.

10. A water trough is in the shape of a right triangular prism with base area of 1250 cm^2 and a height of 100 cm. How much water can be put in before it overflows?



11. José is having vegetable soup. The area of the base of the soup can is 10.4 cm^2, and the height is 10 cm. When José opens the can, he sees that the soup comes up to a height of only 9 cm. What volume of soup is in the can?

12. Bill is building a wooden sandbox with a base area of 8 m^2 for his granddaughters. He does not want to order more than 1.5 m^3 of sand to fill it. He has enough wood to build the sandbox up to 0.22 m deep. What is the minimum height he should build the sandbox to allow the sand to be spread evenly? Justify your answer.

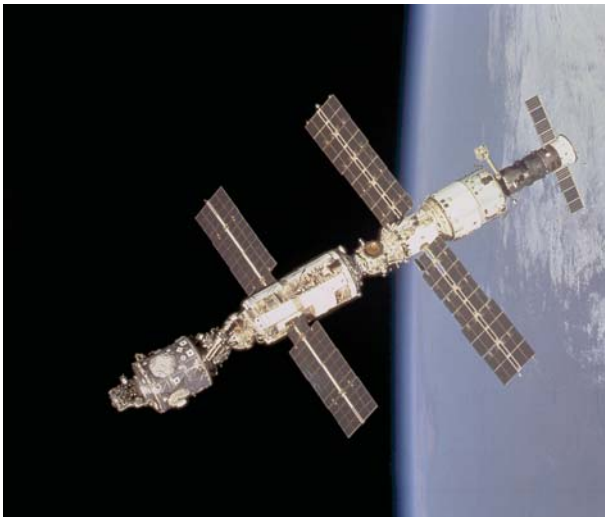
13. Ocean City Aquarium is building a new tank for its coral reef fish. The area of the base is 18 750 cm^2 and the height is 90 cm.

a) What is the volume of the tank in cubic centimetres?

b) What is the volume in litres?

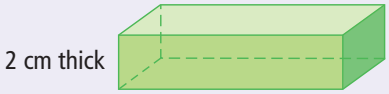
1 L = 1000 cm^3

14. One of the solar arrays on the International Space Station is a rectangular prism with a base area of 892 m^2 and a thickness of 27.5 m. What is the volume of one solar array?



Literacy Link

The word *thick* is sometimes used to describe the height of an object.



15. The International Space Station is shaped like a cylinder that has a cross-sectional area of 615 m^2 and a length of 44.5 m. The living space for the astronauts is 425 m^3. What percent of the volume of the space station is used for living?

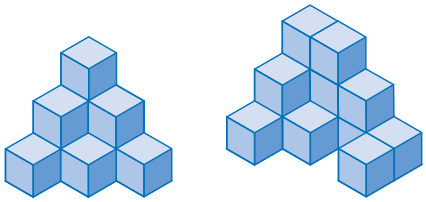
The cross-sectional area is the area of the circle you see if you cut across the cylinder.

WWW Web Link

To learn more about the International Space Station, go to www.mathlinks8.ca and follow the links.

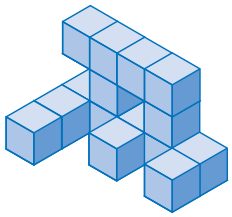
Extend

16. In the structures below, each small cube has a base area of 4 cm^2 and a height of 2 cm. In the first two structures, assume the side facing away from you is solid.



Structure 1

Structure 2



Structure 3

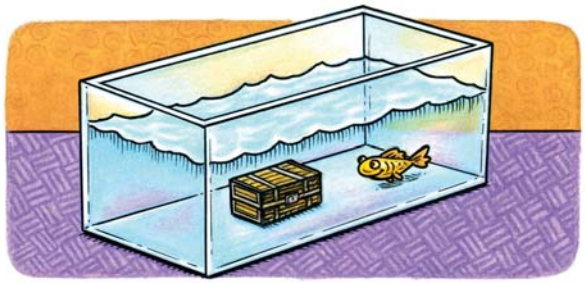
a) How many cubes are in each structure?

b) What is the least number of small cubes needed to complete each structure so that it becomes a rectangular prism?

c) What is the total number of cubes in each completed structure?

d) What is the volume of each completed rectangular prism?

17. Callie's rectangular fish tank has a base area of 800 cm^2 and contains water to a depth of 15 cm. She adds a solid decoration in the shape of a rectangular prism to the bottom of the tank. The decoration has a base area of 40 cm^2 and a height of 5 cm. What is the new level of water in the tank?



18. A cube with a base area of 4 cm^2 and a height of 2 cm is inside a box with a base area of 16 cm^2 and a height of 4 cm.

a) What is the ratio of the volume of the cube to the volume of the box?

b) What is the ratio of the area of the base of the cube to the area of the base of the box?

c) What is the ratio of the height of the cube to the height of the box?

d) What relationship exists among these three ratios?

MATH LINK

Some parks have shelters around the eating areas. These shelters consist of two or three walls. The area of the end of each wall is 0.48 m^2.

a) Sketch and label the dimensions of a sheltered eating area. Keep in mind that the picnic table that will go inside is about 1.8 m long and 0.74 m wide.

b) Calculate the volume of concrete used to make the walls.