

7.3

Volume of a Cylinder

Focus on...

After this lesson, you will be able to...

- determine the volume of a cylinder

Did You Know?


On average, each person in Canadian municipalities uses 604 L of water every day.

How much water do you use? You might be surprised. The water storage tank shown has a height of about 21.6 m and a diameter of about 10.5 m. If the tank is completely filled, predict how long the water would last in your city or town.



Explore the Math

Materials

- centimetre grid paper
-  a variety of empty cylindrical cans
- ruler
- measuring cups
- sand or rice

How can you use area to develop a formula for the volume of a cylinder?

Work with a partner.

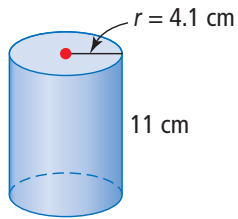
1. Choose a can. Estimate the volume of your can.
2. Calculate the volume of your can. Explain your method.
3. Share your results with three other groups. Compare the estimated and calculated volumes for each cylinder to verify the methods used.

Reflect on Your Findings

4. If you know only the radius of the base and the height of a cylinder, how can you determine the volume of the cylinder? Explain and verify your formula using an example.

Example 1: Determine the Volume of a Cylinder Given the Radius

- a) Estimate the volume of the cylinder.



- b) Calculate the volume of the cylinder. Express your answer to the nearest tenth of a cubic centimetre.

Solution

- a) Use 10 as an approximate value for the height of the cylinder.
Use 4 as an approximate value for the radius of the circular base.

Volume of a cylinder = Area of circular base \times height of cylinder

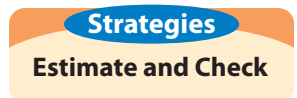
$$\begin{aligned}V &= (\pi \times r^2) \times h \\V &\approx (3 \times 4^2) \times 10 \\V &\approx 3 \times 16 \times 10 \\V &\approx 48 \times 10 \\V &\approx 480\end{aligned}$$

An estimate for the volume of the cylinder is 480 cm^3 .

- b) Volume of a cylinder = Area of circular base \times height of cylinder

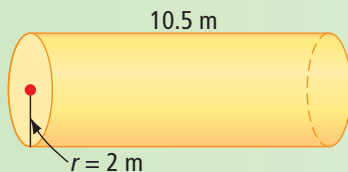
$$\begin{aligned}V &= (\pi \times r^2) \times h \\V &\approx (3.14 \times 4.1^2) \times 11 \\V &\approx 580.6174\end{aligned}$$

The volume of the cylinder is 580.6 cm^3 to the nearest tenth of a cubic centimetre.



Show You Know

- a) Estimate the volume of the cylinder.



- b) Calculate the volume of the cylinder.

Did You Know?

Vulcanization refers to a curing process for rubber that involves high heat and the addition of sulfur.

Example 2: Determine the Volume of a Cylinder Given the Diameter

A hockey puck is made of vulcanized rubber. What is the volume of rubber required to manufacture one puck? Express your answer to the nearest cubic centimetre.



Solution

The diameter is 7.6 cm.

$$\begin{aligned} r &= 7.6 \div 2 \\ &= 3.8 \end{aligned}$$

The radius is 3.8 cm.

Volume = Area of base \times height

$$V = (\pi \times r^2) \times h$$

$$V \approx (3.14 \times 3.8^2) \times 2.5$$

$$V \approx 113.354$$

The volume of the hockey puck is 113 cm³ to the nearest cubic centimetre.

How is the radius related to the diameter?

Show You Know

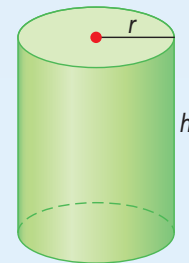
What volume of recyclable waste will fit into one of these bins?



Key Ideas

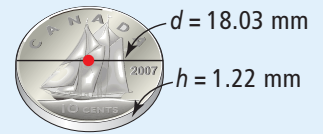
- The base of a cylinder is a circle. The formula for the area of the base of a cylinder is $A = \pi \times r^2$.
- The volume of a cylinder can be found using the formula:
Volume = Area of base \times height

$$V = (\pi \times r^2) \times h$$

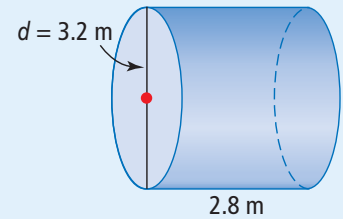


Communicate the Ideas

- List the steps you would use to find the volume of this dime.
 - What information would you need to calculate the volume of a roll of dimes?



- Hanna wants to calculate the volume of the cylinder shown but she does not know which measurement is the height.
 - What is the height?
 - Explain to Hanna how you know this is the height.



- Jethro calculated the volume of the cylinder shown.

$$V = (\pi \times r^2) \times h$$

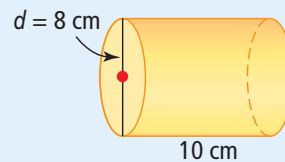
$$V \approx (3.14 \times 8^2) \times 10$$

$$V \approx 3.14 \times 64 \times 10$$

$$V \approx 2009.6$$

The volume of the cylinder is 2009.6 cm^3 .

Has he made an error in his solution? Explain how you know.

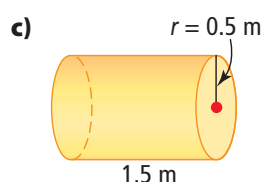
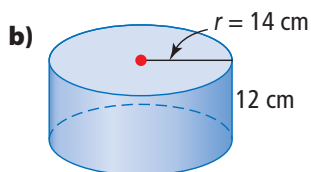
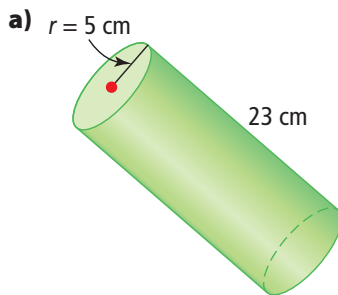


Check Your Understanding

Practise

For help with #4 to #5, refer to Example 1 on page 263.

- Determine the volume of each cylinder.

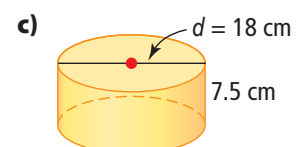
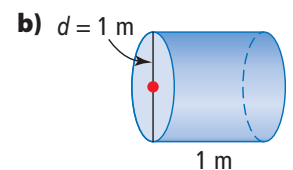
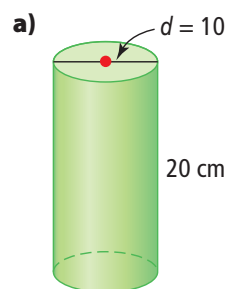


- What is the volume of each cylinder?

- radius = 5 cm, height = 8 cm
- radius = 11 cm, height = 11 cm
- radius = 1.1 m, height = 2.6 m
- height = 25 cm, radius = 4.5 cm

For help with #6 to #7, refer to Example 2 on page 264.

- Determine the volume of each cylinder.



7. What is the volume of each cylinder?
- diameter = 8 cm, height = 12 cm
 - height = 7 m, diameter = 2 m
 - height = 37.5 cm, diameter = 12 cm
 - diameter = 4.5 m, height = 19.5 m

Apply

8. The volume of a cylinder is 150 cm^3 and the area of its base is 48 cm^2 . What is the height to the nearest centimetre?
9. The Canadarm has a cylinder called a *capture envelope* that is used to catch objects floating in space. The capture envelope is 20.3 cm in diameter and 10 cm deep. What is the maximum volume of the capture envelope?



WWW Web Link

To learn more about the Canadarm, go to www.mathlinks8.ca and follow the links.

10. As of early 2006, the International Space Station consisted of several cylindrical elements.

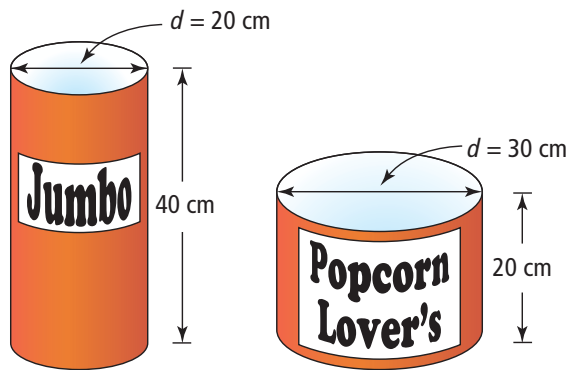
| Element | Length (m) | Diameter (m) |
|-----------------------|------------|--------------|
| Zarya FGB | 12.6 | 4.1 |
| Unity Node 1 | 5.5 | 4.6 |
| Zvezda service module | 13.1 | 4.2 |
| Z1 Truss | 4.9 | 4.2 |
| P6 Truss solar array | 73.2 | 10.7 |
| Destiny | 8.5 | 4.3 |

- Which element has the greatest volume? What is its volume?
- Estimate and calculate the total volume of the International Space Station.

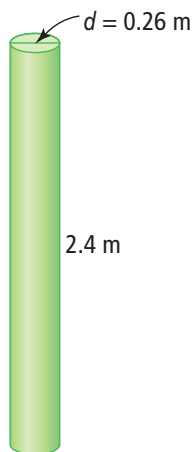
Science 8 Link

The International Space Station takes about 92 min to orbit Earth once.

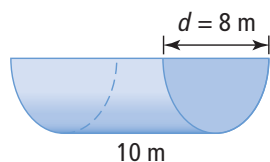
11. Some of the largest drill pipes used in extracting oil have a length of 20 m and an inside diameter of 0.5 m. As oil flows through such a pipe, what is the maximum volume of oil in one pipe at any given time?
12. Martha has a choice of two different popcorn containers at a movie. Both containers are the same price. Which container should Martha buy if she wants more popcorn for her money? Explain.



13. A company uses cardboard tubes like the one shown to make concrete posts for the foundation of a building. If a building requires 35 tubes, what is the volume of concrete required? Give your answer to the next highest cubic metre to make sure that there is enough concrete.



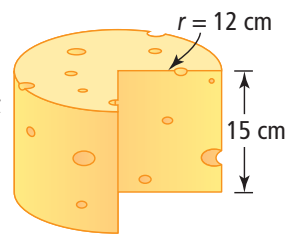
14. Determine the volume of the semi-circular trough.



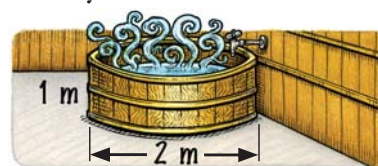
Extend

15. a) What happens to the volume of a cylinder when its radius is doubled? Show how you know.
 b) What happens to the volume of a cylinder when its height is doubled? Show how you know.

16. A piece of cheese was cut from a cylindrical block of cheddar. What volume of cheese was cut from the block? What assumptions did you make?



17. Some Japanese bathtubs are in the shape of a cylinder.



- a) Calculate the volume of water if the tub is filled to a depth of 0.6 m.
 b) If the volume of water is 1.256 m^3 , how deep is the water?
 c) If the water is already 0.5 m deep, how much more water is needed to fill it to a depth of 0.7 m?
18. A cylindrical water storage tank has a height of 21.6 m and a diameter of 10.5 m. If the tank is completely full, how long would the water last in a community of 10 000 people? Assume the average daily water use in the community is 604 L per person. Give your answer to the nearest hour.

$1000 \text{ L} = 1 \text{ m}^3$

MATH LINK

Picnic tables can have either a circular or a rectangular concrete top. The top is held up by a solid concrete column in the shape of a cylinder.

- a) Design two concrete tables.
- Each table has a column with a diameter of 60 cm and a height of your choice.
 - One table must have a circular top. The other table must have a rectangular top.
 - The tabletops cannot exceed a thickness of 10 cm.
- b) Determine the volume of concrete needed to make both picnic tables. Show your calculations.

